A QRS-system (Barton, 1999) is a meaningful system that for a given group of people gives meaning to Quantity, Relations and Space. Each cultural group has their own QRS-system. This paper focuses on how the Sámi culture’s elements lavvu, lasso, skis and duodji can function as basis for mathematics teaching. The elements are described and mathematised (Freudenthal, 1973); made accessible to mathematics treatment. The elements are further categorised into two groups, Relations and Space. There will be pointed out possibilities for how to use these elements in mathematics teaching.

INTRODUCTION
This paper aims to mathematise some elements of the Sámi culture and to build up a framework for analysing these elements. The research question is

How can elements in the Sámi culture be mathematised with respect to Relations and Space in a QRS-system?

In another study (Fyhn, 2004) 10 – 12 years old students from Northern Norway made a two day winter trip to the mountains. The students went skiing to get to the camp area, they spent one night in a lavvu and they performed some activity with a lasso. This trip is referred to as the pilot study in this paper.

BACKGROUND
In my master thesis (Fyhn, 2000) I questioned if students who participate regularly in common leisure time activities succeed in some common mathematics tasks. There could be some connections. Girls who work with creative craft seemed to succeed best in tasks that concerned patterns. In addition students who take part in physical activities seemed to succeed in tasks that concerned understanding of space. Barton’s (1999) R and S from the QRS-system was a suitable way of describing these findings. What I denoted as “patterns” was denoted as “relations” by Barton. Thus I wanted to explore the QRS-system and how it could work out as a tool in my further work.

Barton (2005) points out that bilingual learners who are fluent in both languages have a deeper and more aware sort of knowing mathematics than those who only master one language. The Norwegian curriculum for the 10 year compulsory school (KUF, 1996) was implemented in 1997. This curriculum claims

Sami culture and social life are important parts of the common cultural heritage, which all pupils in compulsory school should learn about. The Sami culture, language, history and social life comprise part of the common content of the different subjects.

The education of Sami pupils shall promote their roots and security in their own culture and to develop the Sami language and Sami identity. At the same time it shall ensure that
Sami pupils are able to participate actively in the community and obtaining education at all levels. (p. 65)

The mathematics teaching for Sámi students today make use of ordinary Norwegian textbooks translated into Sámi language. This is exactly what Barton (2005) warns against; just that one knowledge system has had enormously amounts of time and energy put into its development does not make it the only and correctly structured knowledge. Some mathematics teaching material is designed for Sámi students, for instance Nystad’s booklets which are printed in both Sámi and Norwegian language (Nystad, 2002a, 2002b, 2003). Still there is a long way to go before the curriculum’s (KUF, 1996) intentions are implemented in Norwegian schools.

MATHEMATICS

According to Lakoff and Núñez (2000) mathematics is a product of the human mind. They further claim (p. 6) “Metaphors are an essential part of mathematical thought, not just auxiliary mechanisms used for visualization or ease of understanding.” Patterns in snow are examples of such metaphors. Some Sámi mittens have the pattern *grouse footprints* on their edges. These repeating patterns show a connection between embodied experiences and ornamentation. Ski trails as well as animal footprints perform patterns in the snow, patterns with different symmetry properties (Fyhn, 2002a).

Barton (2005, p. 100) points out that “Rather than thinking of the mathematics which is known world over through formal education, we need to expand our vision to include any form of quantitative, relational or spatial systems.” Barton uses boats as a metaphor for mathematics. He claims that different boats can be used for different purposes, the fishing boat can go to rocky places where the ferry cannot navigate and the ferry can travel under conditions too hard for the fishing boat. “It is the same world, but it is a different understanding. Neither is the truth.” (p. 100)

In this paper I intend to denote geometry and algebra like Lakoff and Núñez (2000) denote the Platonic mathematics – if it exists at all; “a disembodied mathematics which is transcending all bodies and minds and structuring both this universe and every possible universe” (p. 1). When it comes to Duodji I face the problem of whether some patterned ornamentation is geometry or algebra or both. As far as I can see this is an example of two different understandings of the same phenomenon, and neither is the truth.

Hans Freudenthal (1973) restricts geometry and does not include geometrical patterns:

A pragmatic program for geometry could remain restricted to a small treasure of theorems like the Pythagorean, a few obvious theorems on similar figures and a few formulae for perimeters, areas and volumes. (p. 406)

Freudenthal strongly problematises the distinction between geometry and algebra: “Geometrical algebra, … was the disease which killed Greek mathematics” (p. 5). He
continues: “The first to cut free from the Greek tradition was Descartes, the challenger of all tradition. He put the chart before the horse: rather than geometrizing algebra, he algebraized geometry” (p. 6). To me these statements are related to the question about whether “The one and only Mathematics” exists. Barton (2005, p. 98) asks “...why should we think that mathematics is the single universal created from human experience?”

S AND R IN A QRS-SYSTEM

According to Hans Freudenthal (1973) geometry is grasping space.

And since it is about the education of children, it is grasping that space in which the child lives, breathes and moves. The space that the child must learn to know, explore, conquer, in order to live, breathe and move better in it. (p. 403)

Lakoff and Núñez (2000) support this by claiming that prior to the mid-nineteenth century space was conceptualized as most people normally think of it – namely naturally continuous. “It arises because we have a body and a brain and we function in the everyday world” (p. 265).

This work focuses on Space like the students experience it in their daily lives, geometry as grasping space; space as the living arena where geometry activities take place.

When a three dimensional figure from the real world is scaled down and transformed into a two dimensional figure on paper, it is far from as concrete as its origin. School mathematics has traditionally made use of such drawn images instead of making use of the original figures themselves. Berthelot and Salin (1998) divide the space into three different ones: The micro space where school geometry usually takes place, the meso space where the students’ physical and social interactions usually take place and finally the big macro space which contains the whole mountain/forest/city.

According to Berthelot and Salin (1998) one of the main sources of learning difficulties in geometry among students 12 years and older, is probably the previous treatment of geometrical figures on paper during elementary school. In the teaching process, many students think of geometrical figures as if they were objects, while teachers refer to the same figures talking about geometrical concepts. In addition, Berthelot and Salin claim they have good reason to expect that geometrical knowledge is not spontaneously transferred to solve space problems. Berthelot and Salin are here interpreted to support geometry teaching based on the mathematising of space activities.

Different more or less advanced plaited wool ribbons are elements of Duodji. If you plait your hair you split it into three equal parts. This plaiting procedure can be described by numerous repetitions of “take the right part and cross it over the mid-part. Then take the left part and cross it over the mid part”. The right part, whichever it is, can refer to all of the parts of the hair, and so is for the mid part and the left part.
as well. This is what we understand with conceptual metonymy (Lakoff & Núñez, 2000) and it exists outside mathematics.

This everyday conceptual metonymy …plays a major role in mathematical thinking: It allows us to go from concrete (case by case) arithmetic to general algebraic thinking… This everyday cognitive mechanism allows us to state general laws like “x + y = y + x”, which says that adding a number y to another number x yields the same result as adding x to y. It is this metonymic mechanism that makes the discipline of algebra possible, by allowing us to reason about numbers or other entities without knowing which particular entities we are talking about. (p. 74-75)

An analogy for conceptual metonymy in music is the pitch when two or more persons would sing a song together. The structure of the song is given on beforehand; independent of what particular pitch to use. Lakoff and Núñez further claim that “Algebra is the study of mathematical form or ‘structure’” (p. 110). According to The Penguin English Dictionary (Allen, 2002) a synonym for “structure” is “interrelation of elements”. In other words - how elements are related to each other.

I interpret the structured process of working with concrete patterned ornamentation as conceptual metonymy. In this paper I focus on the Relations that concern the structure of concrete and visual patterns; the metonymy in Duodji ornamentation.

**SOME PARTICULAR ELEMENTS IN THE SÁMI CULTURE**

The Sámi tent lavvu looks like the Native Americans’ tipi. Inside it has a central fireplace and its stable structure bears the strong winds that occur in the Scandinavian mountain plateaus. The lasso is a tool for catching running reindeers. In the Sámi skiing championship there is a biathlon discipline; a lasso-ski competition. For youths of twelve and thirteen the distance they throw the lasso is about half a lasso length. The target is a reindeer’s horns, fastened to a stake that is stuck into the snow. The lasso is used by more cultures than the Sámi. But neither the lasso nor the lavvu traditionally belongs to other Scandinavian cultures than the Sámi.

Skiing activities is an element in the Sámi culture (Birkely, 1994). Skis are traditionally used in the rest of Scandinavia, too, as the ground is covered with snow about half of the year in many Sámi areas. According to Birkely the Bysantic historian Prokopios wrote about Thule (Scandinavia) about 1500 years ago. Among the people of Thule Prokopios describes “Skrithifinoi” which means skridfinnar; skiing Sámi people. Birkely claims there are reasons to believe that Sondre Norheim hundreds of years later could have learned his telemark technique from the Sámi people at the Hardangervidda in southern Norway.

The lavvu, the lasso and the skis were all natural belonging elements of the mountain trip in the pilot study. These three elements were naturally connected to activities which took place in Space. Because the trip took place in Space there was no focus on Relations. However, based on Fyhn’s (2000) findings there are reasons to believe that Relations would come to surface by mathematising the patterned ornamentations of duodji.
“Duodji” is a collective term for various activities such as homecrafts, artware (handicrafts), woodwork and trades, and for many Saamis it is a way of life… “Duodji” mirrors the Saami mode of living and cultural traditions. (Aagård, 1994, p. 2)

According to Dunfjeld (2001) duodji includes the process going from idea until the complete product and it is a common concept for Sámi art and handicraft in Norway, Sweden and Finland.

METHOD

The intention of this paper is to build up a framework for analysing mathematics content of some elements in the Sámi culture. In the pilot study the students were videotaped performing activities and some of their discussions were recorded on cassette as well. These videos were initially meant as basis for further tasks in mathematics; tasks that focused on concept understanding. The analyses of this paper concerned the natural use of lavvu, lasso and skis in the tapes and videotapes: What – if any - is the mathematics in these activities which take place in “the naturally continuous space” (Lakoff & Núñez, 2000). Patterned ornamentations do not necessarily take place in the meso space (Berthelot & Salin, 1999), but many people claim this is geometry. Thus I wanted to investigate some patterned ornamentation from Duodji in order to bring in an element that differed from lavvu, lasso and skis.

According to Freudenthal (1991) mathematising includes both moving from the world of life to the world of symbols and deeper diving into the world of symbols. In mathematising the tapes and the videotapes I categorised the mathematics into $R$ and $S$ and related this mathematics to what I believed was geometry and algebra. Here I was influenced by my Norwegian mother tongue. This paper does not focus on the Sámi language, but Barton (2005) warns that ideas of quantity, space and relations are not necessarily expressed the same way in different languages. Further work based on this paper could be to let teachers and students with Sámi language as mother tongue mathematise these four elements. One question will then be how their mathematising differs from mine.

Because Duodji is a genuine Sámi tradition and because the ornamental Duodji patterns are structured patterns, I open for the possibility that Duodji ornamentation can function as a gate to understanding Relations. This will be discussed because ornamentations often are denoted as geometry and thus do not concern algebra.

MATHEMATISING OF SOME ELEMENTS IN THE SÁMI CULTURE

The Lavvu

The lavvu is recognised by its conical shape, it represents a geometrical figure. The central fireplace with people around it represents the centre of a circle and the people around it can represent points on the periphery. I claim that the lavvu’s mathematics mainly belongs to $S$. A description of the lavvu will include usage of some geometrical concepts; circumference, point, area, periphery, triangle, cone and sector.
In the pilot project some questions were asked in order to let the students focus on why the lavvu shape is useful compared to a tent. This showed to be a much too difficult discussion for the students. This was an example of the researcher’s own joy caused by her mathematising of the lavvu. I was trapped just the way many teachers are trapped: “When you first have done mathematising like this, the mathematics seem so obvious that it is a great risk to forget that other people do not see it at all.”

Sitting inside the lavvu the students were capable to watch the floor and make a good estimate about how many people who could sleep in there.

A lavvu that is pitched up can be used as a basis for a discussion about the relation between radius and circumference in a circle (Fyhn, 2004). It is not obvious to the students that this can be investigated by checking if there is any relation between the number of steps from the centre of the lavvu’s fireplace to its doorway and the number of steps in the lavvu’s circumference. This investigation takes place in meso space (Berthelot & Salin, 1998).

Freudenthal (1991) claims that reflection on one’s own activities is an important aspect of mathematising. During my reflections after the pilot research, I found many possibilities for how to use the lavvu in mathematics teaching. In raising the rods of a lavvu the students make use of the knowledge that a right cone has its top straight above its centre and that all the rods are of equal length. The lavvu is centred round its fireplace. On the ground the distance from the fireplace and to the lavvu’s edge is the same all over the lavvu.

One possibility for a further project is to let the students raise a lavvu – that is an activity which takes place in the meso space. Sitting inside the lavvu afterwards the students can then discuss and describe as many as possible of the lavvu’s properties and connect these properties to mathematics.

*The Lasso*

For the lasso to catch a target, there are two conditions that need to be fulfilled. First the distance need to be correct and second the direction must be appropriate. Given the correct distance, the locus of the possible catching points is a circle with the lassoer in its centre. Given a fixed direction, the possible catching points constitute a ray from the lassoer and in the fixed direction. The intersection of the circle and the ray represents that both conditions are fulfilled; the lasso catches its target.

Based on this I will claim that the mathematics of the lasso as element in the Sámi culture mainly belongs to S. In other words: some geometry teaching can be based on the use of lasso. More precisely the geometry teaching that concerns the concepts; circle, angle, point and ray.

In the pilot study the students were divided into groups which took part in some outdoor activities. One of the activities started with letting the group find the middle of a lasso. One group member stood still and held on to this mid point. Another group member held on to the lasso’s ends and walked around person number one with the
lasso straightened. The walking had to be repeated until the footprints shaped a circle’s periphery. The complete lasso was straightened out again and the students were asked how many lasso lengths were around the complete circle periphery. They guessed and checked and all the groups concluded the answer should be somewhere between 3 and 3.5. Some of the students even discussed if the correct answer was 3.2. Afterwards one of the group members stood in the circle’s centre and the other ones on the periphery. The one in the middle then tried to catch the others by the lasso.

Because 3.14 or π not was suggested as answer to the lasso task, I took for granted that the students were not introduced to π yet. Next day at school I presented the students for some tasks in their book that concerned relation between diameter and circumference in a circle. Some of the girls quickly replied that they had done these tasks last week and they could not find any reason to do the same tasks once more. This showed quite clearly that to these students, as for many other Norwegian students as well, mathematics is about finishing tasks in a book, not necessarily to understand what these tasks are all about. Some of the girls that more than once proved to be able to do smart reasoning in mathematics did not see any connections between the lasso task and the similar tasks in the book. This supports Berthelot and Salin’s (1998) claim that geometrical knowledge is not spontaneously transferred to solve space problems.

One possibility for a further project is to let the students use a lasso. Afterwards they can list up properties of a lasso and describe how the lasso is used. Then they may connect the listed properties with what they believe can be mathematics.

The Skis

One of the parents transported the lavvus and most of the other equipment by snow scooter to get it to the camp area. The students used skis to get there. I did not focus on mathematising the students’ skiing. The following day at school the students could have described how they got to and from the camp. They used several different skiing techniques which all resulted in symmetric patterns in the snow.

The students performed some tasks by the use of skis, tasks that I had prepared beforehand. These were not tasks that concerned mathematising; the students just concretised some mathematics by use of their skis. The intention was to do some mathematising but I realised afterwards that I got caught by traditional “school reasoning”. I started to focus on the mathematics concepts instead of focusing on the skis to be mathematised. This reminds me how difficult it is, to me as well as to other teachers, to change the way you teach; to put away some of one’s own old teaching experiences in preparing and implementing teaching.

One skiing task started this way: The students should stand on their skis and turn around while the back ends of their skis were located on the same place all of the time. This activity is a bit difficult to perform and it is an activity with focus on mastering the skis. The result of this task is that you make a nice circle with radius equal to one ski’s length. The centre of this circle is the place where the back ends of
the skis were located and the circumference of the circle consists of the places touched by the skis’ tips. The trampled area represents the area of this circle. The PISA test (Kjærnsli, Lie, Olsen, Roe, & Turmo, 2004) showed that space and shape is the area where Norwegian students perform lowest and that the Norwegian results in this field even have decreased from 2000 until 2003. Thus there are reasons to believe that area is a difficult concept for Norwegian students to understand. However, the PISA test was not translated into Sámi language thus Norwegian students with Sámi as mother tongue did not take part in this study.

One possibility for a further project is to let the students use skis in meso space (Berthelot & Salin, 1998) in a way they choose themselves. Afterwards they can list up properties of skis, how skis are used and then describe what all these have to do with mathematics.

**Duodji**

Regarding Duodji, this paper’s focus is limited to patterned ornamentations. Such structured ornamentations have lots of inter-Relations between their elements; thus they are categorised into $R$. The structured work with these patterns includes metonymy and thus gives possibilities for the work with algebra in school.

In Dunfjeld’s (2001) descriptions of ornaments she, to a large extent, refers to how geometrical figures and other ornaments are structured and why they are structured the way they are. A superficial reading of this text can lead to claiming it is about geometry but I will claim it mainly concerns structure - Relations. An interesting approach to research on this area could be a more profound analysis of the mathematics in this text.

Patterns usually have names, for instance the pattern *fish bone* is well known. This pattern has got its name from the backbone and ribs of a fish. A similar pattern is made when a skier goes uphill, and both the skiing technique for this ascending and the skiing pattern is named fish bone. This fish bone metaphor is what Lakoff and Núñez (2000) denotes as a conceptual metaphor: “Conceptual metaphor is a cognitive mechanism for allowing us to reason about one kind of thing as if it were another” (p. 6). Dunfjeld (2001) chose to interpret the ornaments by the metaphor and the embodied understanding she had made from the ornament. She states: “To watch ornaments just as decoration was not usual” (p. 118, my translation). Dunfjeld further claims that “the ornamentation is ambiguous dependent on which context it is interpreted or analysed into” (p. 39, my translation). This tells me that I must be careful in mathematising Duodji. I cannot avoid doing it because I usually mathematise much of what I experience, but I have to be aware of these close connections between Duodji and context.

Duodji patterns often have names related to nature. This indicates an embodied way of using metaphors. The pattern for sewing a coffee bag consists of different parts and each part has its own name like for instance “njálbmi”, the part where the opening is (Johansen, 1983). The Sámi word “njálbmi” means “mouth” in English.
However, this pattern is an ordinary geometrical figure and not a patterned ornamentation.

Mathematising (Freudenthal, 1973, 1991) some particular Duodji ornamentation could start with pointing out how an embodied metaphor is used as name for this ornamentation. For instance the grouse footprints refer to the similarity to the footprints grouses make in the snow while they are walking around and eating. It is a Sámi tradition to catch grouses in snares during the winter time and the birds’ footprints tell where to put up the snares. These repeated footprints shape an ornamented pattern. Knitting of this pattern can be described by use of the conceptual metonymy; first turn: three times bottom colour, second turn: one bottom colour, one different colour and one bottom colour. The third turn equals to the first one, the second turn equals to the second one and so on. According to Lakoff and Núñez (2000) this is algebra.

As for the QRS-system the description of this knitted pattern to a large extent concern Relations.

**A FRAMEWORK FOR ANALYSES OF CULTURAL ELEMENTS**

Freudenthal (1973) claims that geometry is grasping space. Space is an important part of mathematics. Both Barton (1999) and Lakoff and Núñez (2000) make use of the term space in their descriptions of mathematics. So do I (Fyhn, 2000).

Lakoff and Núñez (2000) use the terms essence and structure in their approach to algebra,

*Algebra is about essence. It makes use of the same metaphor for essence that Plato did – namely, Essence is form.*

*Algebra is the study of mathematical form or “structure”. Since form (as the Greek philosophers assumed) is taken to be abstract, algebra is about abstract structure.* (p. 110)

Fyhn (2000) used the metaphor pattern quite similar to Lakoff and Núñez’ structure. Barton’s (1999) term Relations points to some extent at algebra, but it seems to be a wider concept than structure.

Descartes introduced the world to the metaphor “space-is-a–set-of-points” (Lakoff, & Núñez, 2000). Barton’s (1999) QRS-system can be visualized by the following: “Let (q, r, s) be an ordered triple of non-negative real numbers. Then the QRS-system is the space which is generated by (q, r, s).” Each of q, r and s can be described as basic gates of mathematics understanding. Most people have their understanding and interpretations of different kinds of mathematics somewhere in this space. These close connections between the Q, R and S make it difficult to categorise some particular mathematics as just Q or R or S.

Tables 1-2 show a framework for analyses of lasso and lavvu with respect to R and S. Just some aspects of these analysed objects are shown; the intention is to present the framework, not to fulfil these analyses.
relations | space
---|---
graphy | The connection between diameter and circumference
Concept building: Properties of the lasso, the circle, the segment. The concepts distance and direction
algebra | $\pi r^2$, $2\pi r$ and other formulas

Table 1. A framework for analyses of lasso

| relations | space |
---|---|
graphy | The location of the fireplace and its relation to the length of the rods and the location of the top point. Different locations inside the lavvu
Concept building: Properties of the lavvu, the rods, the lavvu cloth, the cone, the circle, the sector
algebra | $V = \pi r^2 h$, $r^2 + h^2 = (\text{rod})^2$ and other formulas

Table 2. A framework for analyses of lavvu

SUMMARY AND CONCLUSION
The pilot study in this research here points at an example of what Berthelot and Salin (1998) claim; geometrical knowledge is not spontaneously transferred to solve space problems. Work with the lasso gave reasons to believe that students do not intuitively find connections between their experiences from activities outside the mathematics classroom and tasks in their books. Thus one focus in a possible further research on this issue must be to focus on the process that goes on while the students try to connect their practical experiences to the written tasks in their books. There is a risk that when the teacher once has mathematised an element, its mathematics content becomes so obvious to the teacher that she or he forgets the state they were in before this mathematising took place.

The natural use of the lavvu, the lasso and skis take place in “the naturally continuous space” (Lakoff & Núñez, 2000). Mathematising of these elements mainly categorise them into $S$. The framework shown in the tables 1-2 points at the difference between
understanding of a concept and of a formula as the latter is an algebraic expression for some geometry.

There seems to be no clear distinction between Space and Relations on one side and geometry and algebra on the other. Still I will claim that the use of the distinction points to where the mathematics comes from; Space and Relations focus on the real world while geometry and algebra traditionally focus on the mathematics.

During this work I realised that focusing on concepts of mathematics could prevent me from being open minded towards the outcomes of the mathematising. That is because my view at the Sámi culture is from the outside. I believe this is the greatest challenge to me in a future work in this field.

The structures of the Duodji ornamentations are concrete. Mathematising of the structures in plaiting and weaving leads to permutations which is part of algebra. Based on this a new question arises: Will mathematising of structures in Duodji ornamentations lead to abstract algebra? If so, then mathematising of Duodji could function as a tool to enlighten algebra. It could even be possible that algebra could function as a tool to describe Duodji. This could be an issue for future research.

During this work I have got into more than one discussion about whether ornamented patterns belong to geometry or algebra. My master’s thesis (Fyhn, 2000) gave reasons to believe that as for Space and Relations, students who enjoy taking part in different kind of activities succeed with different kinds of mathematics. By now I know about two ways of mathematising ornamented patterns. My mother tongue is Norwegian which is totally different from Sámi. What I do not know is how people with Sámi mother tongue will mathematise Duodji. Maybe their mathematising will result in one or more new species in the flora of mathematising Duodji ornamentations.

References:


